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Control of Colpitts-Oscillator via Adaptive Feedback Control

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ABSTRACT

This paper investigates the control of chaotic oscillatator, viz. Colpitts chaotic oscillator by adaptive feedbackcontrol method. Our theorem on control for Colpitts oscillatior is established using Lyapunov stability theory. The adaptive feedback scheme links the choice of a Lyapunov function with the design of a controller. The adaptive control is convenient to estimate unknown parameters in chaotic systems. In practical there is no derivative existing in controller part, so it reduces the cost of controller design. Numerical simulations are also given to illustrate and validate the results derived in this paper.

Keywords: Chaos, Adaptive Feedback Control, Colpitts-Oscillator.

1. Introduction

Chaotic dynamical systems are described by nonlinear differential equations and it can be strongly sensitive to initial conditions (Alligood and Yorke, 1997, Baker and Gollub, 1996, Ott, 2002).

Chaos theory has been applied in many fields such as mathematics, computer sciences (Murali and Lakshmaman, 2003), ecology, chemistry (Coffman et al., 1987), physical science(Zhang, 2005) , population dynamics (Rao et al., 2015) and robotics, etc (Blasius et al., 1999, Chen, 2008, Chen et al., 2009, Ghosh et al., 2008, Han et al., 1995, Kakmeni et al., 2006, Kocarev, 1995, Lakshmanam and Murali, 1996, Murali and Lakshmaman, 2003, Ott et al., 1990, Park and Kwon, 2003, Yang et al., 2009, Yang and Cao, 2010, Zhao, 2009) .

In recent years, various control techniques have been deployed to carry control the chaotic systems such as PC method (Pecora and Carroll, 1990, 1991), OGY method (Ott, 2002), sample-data feedback method, sliding mode control method (Che et al., 2010, Ya, 2004), backstepping control method (Suresh and Sundarapandian, 2012a,b,c, Wu and Lu, 2003, Yu and Zhang, 2006), active nonlinear control method (Rasappan et al., 2015, Sundarapandian and Suresh, 2010), delayed feedback control method (Park and Kwon, 2003), etc.

Recently, nonlinear feedback control techniques have been taken much attention in controlling the chaotic systems. In this method, the controller tracks the unstable periodic orbit to stable periodic orbit and estimates the unknown parameters in chaotic systems.

This paper is organized as follows. In section 2, the result for adaptive feedback control system is derived. In section 3, the Colpitts oscillator (Kennedy, 1994) and its application are clearly defined. In section 4, the adaptive feedback control for Colpitts oscillator is derived, the adaptive feedback control is derived using Lyapunov stability theory. The proposed adaptive feedback control is very simple and effective to implement in application sides. Conclusions are contained in final section.

2. Problem Statement and Methodology

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) + \alpha_A + u \qquad (1)$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameter, the matrix A have some unknown parameters, $f\colon R^n \to R^n$ is the nonlinear part of the system, u $\in R^n$ is the adaptive nonlinear feedback controller. α_A is the estimator of unknown parameter.

The global control problem is essentially to find adaptive feedback controller u and $\hat{\alpha}_A$, so as to stabilize the dynamics (1) for all initial conditions $x(0) \in R^n$, i.e.

$$\lim_{t\to\infty} \|x(t)\| = 0$$

for all initial conditions $x(0) \in R^n$ Lyapunov function methodology is used for establishing the adaptive feedback control of the system (1).

By the Lyapunov function methodology, a candidate Lyapunov function is taken as

$$V(x) \quad = \quad x^T P x + \alpha_A^T P_1 \alpha_A \tag{2}$$

where $P,\ P_1$ are $n \times n$ positive definite matrix

Note that $V\colon R^n \to R^n$ is a positive definite function by construction. It is assumed that the parameters of the system (1) are measurable.

If a controller u and $\hat{\alpha}_A$ can be found such that

$$\dot{V}(x) = -x^T Q x - \alpha_A^T Q_1 \alpha_A \tag{3}$$

where $Q,\ Q_1$ are positive definite matrix,then $\dot{V}(x)$ is a negative definite function.

Hence, by Lyapunov stability theory (Hahn, 1967), the dynamics (1) is globally exponentially stable and hence the condition

$$\lim_{t\to\infty} \|x(t)\| = 0$$

will be satisfied for all initial conditions $x(0) \in R^n$.

Then the states of the system (1) will be globally exponentially stable.

3. System Description

Colpitts oscillators are generally used to generate periodical signals. The bipolar junction transistor gives the chaotic nature in Colpitts oscillator (Kennedy, 1994). Colpitts oscillators are using it for lighting the fluorescent lamp and applied to generate a carrier signals at the transmitters.

The state space representation of Colpitts oscillators is

$$\begin{aligned} \dot{x}_1 &= x_2 - f(x_3) \\ \dot{x}_2 &= c - x_1 - bx_2 - x_3 \\ \dot{x}_3 &= x_2 - d \end{aligned} \tag{4}$$

where

$$f(x_3) = \begin{cases} -a(x_3+1) & x_3 \leq -1 \\ 0 & x_3 > -1 \end{cases}$$

and $x_1,\ x_2,\ x_3$ are the state variables and $a,\ b,\ c, d$ are positive constants. The system (4) is chaotic when the parameters are chosen as

$$a = 81.41,\ b = 0.82,\ c = 7.14, \text{and } d = 0.73$$

Figure 3 shows the state orbit of Colpitts-oscilattor.

4. Control of Colpitts- Oscillator via Adaptive Feedback Control

The Colpitts- oscillator dynamics (Kennedy, 1994) is described by

$$\begin{aligned} \dot{x_1} &= x_2 - f(x_3) + u_1 \\ \dot{x_2} &= c - x_1 - bx_2 - x_3 + u_2 \\ \dot{x_3} &= x_2 - d + u_3 \end{aligned} \tag{5}$$

where

$$f(x_3) = \begin{cases} -a(x_3+1) & x_3 \leq -1 \\ 0 & x_3 > -1 \end{cases}$$

and $x_1,\ x_2,\ x_3$ are the state variables and $a,\ b,\ c, d$ are positive unknown constants.
In this paper, we introduce the adaptive feedback procedure to design the controllers u_1, u_2, u_3.

Where u_1, u_2, u_3 is control inputs, which are the function the state variables x_1, x_2, x_3 and unknown estimator $\alpha_a, \alpha_b, \alpha_c, \alpha_d$. As long as these feedbacks stabilize system (5) converge to zero as the time t goes to infinity. That means that, this gives the system (5)

$$\lim_{t\to\infty} \|x(t)\| = 0.$$

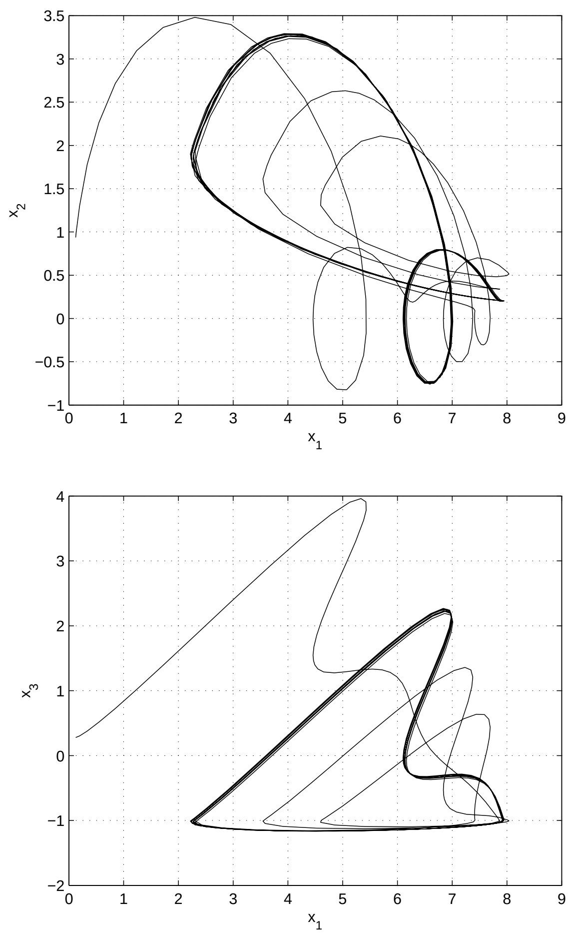

Figure 1: State Orbit of Colpitts- Oscillator

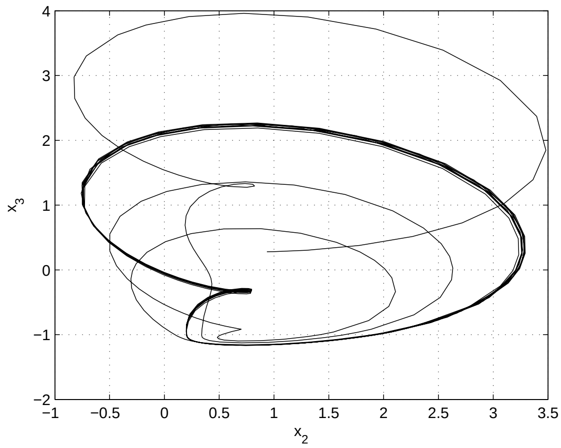

Figure 2: State Orbit of Colpitts- Oscillator

4.1 Case 1 When $x_3 \leq -1$

The dynamics of the collpits oscillator is described by the equation

$$\begin{aligned} \dot{x_1} &= x_2 + a(x_3+1) + u_1 \\ \dot{x_2} &= c - x_1 - bx_2 - x_3 + u_2 \\ \dot{x_3} &= x_2 - d + u_3 \end{aligned} \tag{6}$$

where $x_1,\ x_2,\ x_3$ are the state variables and $a,\ b,\ c, d$ are positive unknown constants.

The candidate Lyapunov function is taken as

$$\begin{aligned} V(x_1, x_2, x_3, \alpha_a, \alpha_b, \alpha_c, \alpha_d) &= \tfrac{1}{2}x_1^2 + \tfrac{1}{2}x_2^2 + \tfrac{1}{2}x_3^2 + \\ &\quad \tfrac{1}{2}\alpha_a^2 + \tfrac{1}{2}\alpha_b^2 + \tfrac{1}{2}\alpha_c^2 + \tfrac{1}{2}\alpha_d^2 \end{aligned} \tag{7}$$

Let define the parameter estimation as

$$\alpha_a = a - \hat{a},\ \alpha_b = b - \hat{b},\ \alpha_c = c - \hat{c},\ \alpha_d = d - \hat{d} \tag{8}$$

Differentiating (7) along the trajectories of the system (6) and (8), the simple calculation gives

$$\begin{aligned}\dot{V} &= x_1(x_2 + a(x_1+1) + u_1) + x_2(c - x_1 - bx_2 - x_3 + u_2) + x_3(x_2 - d + u_3) \\ &\quad + \alpha_a(-\dot{\hat{\alpha}}_a) + \alpha_b(-\dot{\hat{\alpha}}_b) + \alpha_c(-\dot{\hat{\alpha}}_c) + \alpha_d(-\dot{\hat{\alpha}}_d)\end{aligned} \tag{9}$$

The adaptive feedback control is defined by

$$\begin{aligned} u_1 &= -x_2 - \hat{a}x_3 - \hat{a} - x_1 \\ u_2 &= -\hat{c} + x_1 + \hat{b}x_2 + x_3 - x_2 \\ u_3 &= -x_2 + \hat{d} - x_3 \end{aligned} \tag{10}$$

The parameters are updated by the updating law

$$\begin{aligned} \dot{\hat{\alpha}}_a &= x_1(1 + x_3) + \alpha_a \\ \dot{\hat{\alpha}}_b &= -x_2^2 + \alpha_b \\ \dot{\hat{\alpha}}_c &= x_2 + \alpha_c \\ \dot{\hat{\alpha}}_d &= -x_3 + \alpha_d \end{aligned} \tag{11}$$

Substituting equation (10) and (11) in (9), then its implies that

$$\dot{V} = -x_1^2 - x_2^2 - x_3^2 - \alpha_a^2 - \alpha_b^2 - \alpha_c^2 - \alpha_d^2 \tag{12}$$

which is a negative definite function.

Hence, by Lyapunov stability theory (Hahn, 1967), the Colpitts-oscillator (6) is globally exponentially stable.

Theorem 4.1. *The chaotic Colpitts-oscillator (6) is globally exponentially stable with the adaptive feedback control (10) and the unknown parameter estimator (11).*

4.2 Case 2 when $x_3 < -1$

The dynamics of the collpits oscillator is described by the equation

$$\begin{aligned} \dot{x}_1 &= x_2 + u_1 \\ \dot{x}_2 &= c - x_1 - bx_2 - x_3 + u_2 \\ \dot{x}_3 &= x_2 - d + u_3 \end{aligned} \tag{13}$$

where $x_1,\ x_2,\ x_3$ are the state variables and $a,\ b,\ c,\ d$ are positive unknown constants.

The candidate Lyapunov function is taken as

$$\begin{aligned} V(x_1, x_2, x_3, \alpha_a, \alpha_b, \alpha_c, \alpha_d) &= \tfrac{1}{2}x_1^2 + \tfrac{1}{2}x_2^2 + \tfrac{1}{2}x_3^2 \\ &\quad + \tfrac{1}{2}\alpha_a^2 + \tfrac{1}{2}\alpha_b^2 + \tfrac{1}{2}\alpha_c^2 + \tfrac{1}{2}\alpha_d^2 \end{aligned} \tag{14}$$

Let define the parameter estimation as

$$\alpha_a = a - \hat{a},\ \ \alpha_b = b - \hat{b},\ \ \alpha_c = c - \hat{c},\ \ \alpha_d = d - \hat{d} \tag{15}$$

Differentiating (14) along the trajectories of the system (13) and (15), the simple calculation gives

$$\begin{aligned} \dot{V} &= x_1(x_2 + u_1) + x_2(c - x_1 - bx_2 - x_3 + u_2) + x_3(x_2 - d + u_3) \\ &\quad +\alpha_a(-\dot{\hat{\alpha}}_a) + \alpha_b(-\dot{\hat{\alpha}}_b) + \alpha_c(-\dot{\hat{\alpha}}_c) + \alpha_d(-\dot{\hat{\alpha}}_d) \end{aligned} \tag{16}$$

The adaptive feedback control is defined by

$$\begin{aligned} u_1 &= -x_2 - x_1 \\ u_2 &= -\hat{c} + x_1 + \hat{b}x_2 + x_3 - x_2 \\ u_3 &= -x_2 + \hat{d} - x_3 \end{aligned} \tag{17}$$

The parameters are updated by the updating law

$$\begin{aligned} \dot{\hat{\alpha}}_a &= \alpha_a \\ \dot{\hat{\alpha}}_b &= -x_2^2 + \alpha_b \\ \dot{\hat{\alpha}}_c &= x_2 + \alpha_c \\ \dot{\hat{\alpha}}_d &= -x_3 + \alpha_d \end{aligned} \tag{18}$$

Substituting equation(17) and (18) in (16), then its implies that

$$\dot{V} = -x_1^2 - x_2^2 - x_3^2 - \alpha_a^2 - \alpha_b^2 - \alpha_c^2 - \alpha_d^2 \tag{19}$$

which is a negative definite function.

Hence, by Lyapunov stability theory (Hahn, 1967), the Colpitts-oscillator (6) is globally exponentially stable.

Theorem 4.2. *The chaotic Colpitts-oscillator (13) is globally exponentially stable with the adaptive feedback control (17) and the unknown parameter estimator (18).*

5. NUMERICAL SIMULATION

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the differential equations (5) with the adaptive feedback controls u given by (10) and (17).

The parameters of the Colpitts-oscillators (5) is chosen as

$$a = 81.41, b = 0.82, c = 7.14 \text{and} d = 0.73$$

so that the system are chaotic.

The initial values of the system (5) are chosen as

$$x_1(0) = 24.1, x_2(0) = 15.5, x_3(0) = 16.8$$

The initial values of the estimated parameters are

$$\alpha_a = 5, \alpha_b = 10, \alpha_c = 20, \alpha_d = 30.$$

Figure (3) shows the control Colpitts-oscillators (5).

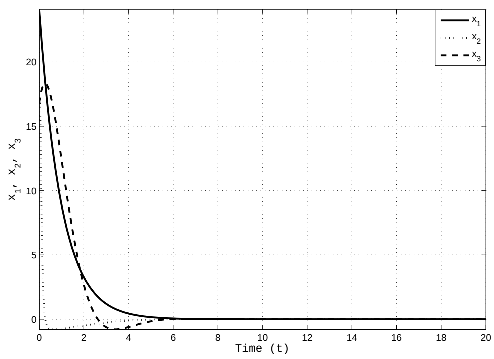

Figure 3: the control of Colpitts-oscillators

Figure (4) describes the unknown estimator of Colpitts-oscillators (5).

6. CONCLUSION

In this paper, adaptive feedback control method has been applied to achieve control the Colpitts chaotic oscillators ([16]). The advantage of this method is a systematic procedure for synchronizing chaotic system and there is no derivative in controller. The adaptive feedback control design has been deployed to

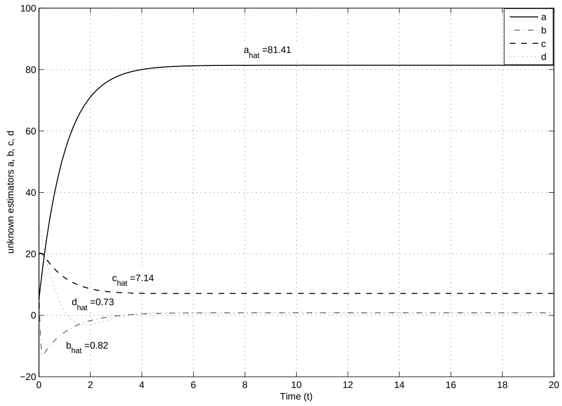

Figure 4: The estimator of the unknown parameters

Colpitts-oscillator. Numerical simulations have been given to illustrate and validate the effectiveness of the proposed control schemes of the chaotic systems.

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